

# SUDOKU X

		8	7		4	9		
	2				5			
	6	5		8		4		
								7
		3				6		
5								
	5	9		6		1	3	
			8				4	
		1	3		9	7		

Moderate

## Samples Document

by

Andrew Stuart

MM Multimedia Ltd

# ABOUT SUDOKU X

This is a variant of the popular Sudoku puzzle which contains two extra constraints on the solution, namely the diagonals, typically indicated by grey cells. In a normal sudoku puzzle all rows, columns and 3x3 boxes must be filled with the numbers 1 to 9 without repeating a number. There are nine 'cells' in every row, column and box.

In Sudoku X the two diagonals containing nine cells and sharing the central grid cell must also be filled with exactly 1 to 9. The puzzle solver can use this information to reduce the possibilities in those lines and make deductions across the board previously out of reach in a normal sudoku. However, these extra constraints allow the puzzle compiler to reduce the number of necessary clues thus creating a balanced puzzle that rivals normal sudoku in variety and difficulty.

Note, it is perfectly possible to create a normal sudoku that co-incidentally has the unique 1 to 9 in each diagonal but unless this information is revealed first it is usually of no help to the solver.

In Sudoku X all the normal sudoku strategies apply - and there are a great number of these. But to complete our puzzles the solver must be expected to use the extra diagonals. This rule is in proportion to the difficulty. Gentle puzzles should require little to no note taking and can often be solved with normal 'eyeballing' techniques. As the grade increases the diagonals contain much more important information.

Some of the basic strategies are illustrated after the examples and show how they can be extended to the diagonals. There are also two examples which show the pitfalls of Uniqueness strategies - and how they do not always apply in the case of Sudoku X.

As with all our puzzles, Andrew Stuart guarantees that there is exactly one solution. Unless explicitly stated they also have a logical solve route, although on the very hard ones this might be obscure to say the least.

To order these puzzles contact **Kate Mepham** on **+44 (0)1373 452277** or email **kate@michaelmepham.com**

For technical, strategy or production related questions, please email **Andrew Stuart** on **andrew@sudoku.org.uk**

Our main web site is  
<http://www.sudoku.org.uk/>

Copyright: The puzzles in this document are strictly the copyright of Andrew Stuart and MM Multimedia Ltd and cannot be used or re-produced outside the purpose of this document which is as a sample. Individuals may copy the puzzles to work sheets in order to solve them. The document may be freely distributed.

2.

	6						3	
		7						
3		9				8		6
	3		5		4		8	
		5				1		
	2		9		7		5	
1		2				4		8
						5		3
	4						7	

Easiest Gentle

score=23

3.

			3	6	1			
			4		9	1	2	5
8				2				4
								9
5								
6				3				
7	9	8	2		6			
			8	5	7			

Typical Gentle

clues=22  
score = 38

4.

3		2		1				4
			3	2	5			
		1			7			
2	6						3	5
		9	8			4		
				9	4			
6			7			2		8

Typical Tough

Hidden Pairs mostly

5.

						9		
3				9	7			8
	2							1
			2					
	1	2	6		3	4	9	
			7		9			
7							1	
			3	1	5			4
		1						

Typical Diabolical

23 clues

6.

	1			3	2			
7								
	5		4		6		8	
	4	9		8		1		6
	6		7		9		2	
								8
				6			5	

Typical Extreme

20 clues  
score=930

7.

8			9					4
7			4		1	2		5
3					9			7
2			8					9
9		3	1		8	4		2
1			7		2			3

Most Extreme

score=2399  
clues = 24

The hardest with  
a known logical  
solve route

8.

				1			8	
						2		
3	5							
		2	3		1	6		
5								7
		4	6		8	3		
								3
		1						
	7							

18 clue Moderate  
Low number of clues

9.

9			5					7
7								1
			1			2		
	1		3					
			7					
					8		6	
		6			9			
2			8					
5					3			

18 clue diabolical  
Low number of clues

10.

				4				
		1		3		6		9
			2		1			
	1	2				5		
	5	6				1		
		3				4	7	
			6		7			
		5				7		

'Worst Unsolvable'

Example unsolvable where the least number of solutions have been found (none)

	1	2	3	4	5	6	7	8	9	
A	2 3 5 6 8 9	2 3 6		5 7 8 9	4	5 6 8 9	2 3 8	1 2 3 5 8	1 2 3 5 7 8	
B	2 5 8	4 7 8	1	5 7 8	3	5 8	6	4 5 8	2 9	
C	3 5 6 8 9	4 7 8 9	3 6 7 8 9	4 7 8 9	2	5 6 7 8 9	1	3 8	4 5 8	3 4 5 7 8
D	4 7	1	2	4 7 8 9	3 6 7 8 9	4 6 8 9	5	3 6 8 9	3 6	
E	4 7	5	6	4 7 8 9	3 2 7 8 9	2 3 4 8 9	1	2 3 8 9	2 3 8	
F			3	1 5	1 2 5 6	2 5 6	4	7	2 6	
G	1 2 3 8 9	4 8 9	2 3 4 8 9	6	1 2 5 8 9	7	2 3 8 9	2 3 4 5 8	2 3 4 5 8	
H	1 2 3 6 8 9	4 8 9	2 3 6 8 9	5	1 3 4 8 9	1 2 8 9	2 3 4 8 9	7	1 2 3 4 6 8	2 3 4 6 8
J	1 2 3 6 8 9	4 8 9	2 3 6 7 8 9	4 8 9	1 3 4 5 8 9	1 2 5 8 9	2 3 4 5 8 9	2 3 8	2 3 4 5 6 8	1 2 3 4 5 6 8

# STRATEGIES

This is by no means exhaustive, but a few pointers to those familiar with sudoku strategies but haven't sat down to think about the implications of Sudoku X.

It is a given to expect Naked and Hidden Pairs, Triples and Quads in the diagonals as you would in any row, column or box.

	1	2	3	4	5	6	7	8	9
A	2	4	5 <sup>3</sup>	1	5 <sup>6</sup> 7 <sup>9</sup>	5 <sup>6</sup> 7 <sup>9</sup>	3 <sup>6</sup> 7 <sup>9</sup>	6 <sup>7</sup> 9 <sup>9</sup>	8
B	7 <sup>8</sup>	6	1 <sup>3</sup>	2 <sup>4</sup> 7 <sup>9</sup>	2 <sup>7</sup> 8 <sup>9</sup>	2 <sup>7</sup> 8 <sup>9</sup>	1 <sup>3</sup> 7 <sup>8</sup>	5	4 <sup>7</sup> 9 <sup>9</sup>
C	7 <sup>8</sup>	9	1 <sup>5</sup>	4 <sup>5</sup> 7 <sup>7</sup>	3 <sup>5</sup> 6 <sup>7</sup>	3 <sup>5</sup> 6 <sup>7</sup>	2	1 <sup>6</sup> 7 <sup>7</sup>	4 <sup>7</sup>
D	4	7	2	8	1	9	5	3	6
E	5	1	6	2 <sup>3</sup>	4	2 <sup>3</sup>	7 <sup>9</sup>	8	7 <sup>9</sup>
F	3	8	9	6	7 <sup>5</sup>	7 <sup>5</sup>	4	2	1
G	6 <sup>9</sup>	5	7	3 <sup>9</sup>	3 <sup>6</sup> 8 <sup>9</sup>	1 <sup>3</sup> 8 <sup>6</sup>	1 <sup>9</sup>	4	2
H	6 <sup>9</sup>	3	4	2 <sup>7</sup> 9 <sup>9</sup>	2 <sup>6</sup> 7 <sup>9</sup>	1 <sup>2</sup> 6 <sup>7</sup>	8	1 <sup>7</sup> 9 <sup>9</sup>	5
J	1	2	8	7 <sup>5</sup> 9 <sup>9</sup>	5 <sup>6</sup> 7 <sup>9</sup>	4	7 <sup>6</sup> 9 <sup>9</sup>	7 <sup>6</sup> 9 <sup>9</sup>	3

## Pointing Pairs

Beyond these are the two types of Intersection Removal. Since the diagonals only ever cross one row or one column at a time the only intersections you should look for are with boxes.

In this example the two yellow cells are the only two cells in box 9 with the candidate 1. These happen to align on the diagonal and since 1 must occur on one of the two yellow cells (we just don't know which one) it can be removed from anywhere else in the diagonal, namely cell C3.

	1	2	3	4	5	6	7	8	9
A	3 <sup>6</sup> 9 <sup>9</sup>	5 <sup>3</sup>	5 <sup>6</sup>	7	8	2 <sup>3</sup> 5 <sup>6</sup> 9 <sup>9</sup>	2 <sup>3</sup> 5 <sup>6</sup>	1	4
B	1 <sup>3</sup> 8 <sup>6</sup>	2	7	1 <sup>5</sup> 6 <sup>9</sup>	4	3 <sup>5</sup> 6 <sup>9</sup>	3 <sup>5</sup> 6 <sup>9</sup>	9	3 <sup>5</sup> 8 <sup>6</sup>
C	1 <sup>3</sup> 8 <sup>6</sup> 9 <sup>9</sup>	4	1 <sup>5</sup> 6 <sup>9</sup>	1 <sup>2</sup> 5 <sup>6</sup> 9 <sup>9</sup>	1 <sup>2</sup> 3 <sup>5</sup> 6 <sup>9</sup>	2 <sup>3</sup> 5 <sup>6</sup> 9 <sup>9</sup>	2 <sup>3</sup> 5 <sup>6</sup>	7	2 <sup>3</sup> 5 <sup>6</sup> 8 <sup>9</sup>
D	5	9	4 <sup>2</sup> 8 <sup>6</sup>	4 <sup>6</sup>	2 <sup>6</sup>	7	1	3	2 <sup>8</sup> 6 <sup>9</sup>
E	4 <sup>2</sup> 8 <sup>6</sup>	7	4 <sup>2</sup> 8 <sup>6</sup>	3	5 <sup>6</sup>	1	2 <sup>6</sup> 8 <sup>9</sup>	4 <sup>5</sup>	2 <sup>6</sup> 8 <sup>9</sup>
F	4 <sup>2</sup> 6 <sup>9</sup>	1	3	8	2 <sup>5</sup> 6 <sup>9</sup>	4 <sup>5</sup> 6 <sup>9</sup>	2 <sup>6</sup> 9 <sup>9</sup>	4 <sup>5</sup>	7
G	1 <sup>2</sup> 3 <sup>4</sup>	8	1 <sup>2</sup> 5 <sup>9</sup>	1 <sup>2</sup> 4 <sup>5</sup>	1 <sup>2</sup> 3 <sup>5</sup>	2 <sup>3</sup> 4 <sup>5</sup> 9 <sup>9</sup>	7	6	1 <sup>5</sup> 3 <sup>9</sup>
H	7	5 <sup>3</sup>	9	1 <sup>2</sup> 5 <sup>6</sup>	1 <sup>2</sup> 3 <sup>5</sup> 6 <sup>9</sup>	2 <sup>3</sup> 5 <sup>6</sup>	4	8	1 <sup>5</sup> 3 <sup>9</sup>
J	1 <sup>3</sup>	6	1 <sup>4</sup> 5 <sup>9</sup>	1 <sup>4</sup> 5 <sup>9</sup>	7	8	5 <sup>3</sup> 9 <sup>9</sup>	2	1 <sup>5</sup> 3 <sup>9</sup>

## Box Line Reduction

The converse of this is Box/Line Reduction as illustrated in the next example. Since all the occurrences of candidate 1 on the diagonal (yellow cells) occur in the same box, none of the other 1s in that box are valid and can be removed. The line reduces the box.

11.

		9		1		3		
					4		9	
			6		3			
9			7					1
6							3	5
1					9			2
			4		6			
	7		2				1	
				7		6		

This Diabolical requires a Simple Colouring strategy at the stage you see below

	1	2	3	4	5	6	7	8	9
A	<sup>2</sup> <sub>5</sub>	4	9	8	1	7	3	<sup>2</sup> <sub>5</sub>	6
B	7	6	3	5	2	4	1	9	8
C	<sup>2</sup> <sub>5</sub> 8	1	<sup>5</sup> <sub>8</sub>	6	9	3	<sup>2</sup> <sub>5</sub>	7	4
D	9	3	<sup>2</sup> <sub>5</sub>	7	<sup>5</sup> <sub>6</sub> 8	<sup>2</sup> <sub>5</sub> 8	4	<sup>6</sup> 8	1
E	6	<sup>2</sup> 8	7	1	4	<sup>2</sup> 8	9	3	5
F	1	<sup>5</sup> 8	4	3	<sup>5</sup> <sub>6</sub> 8	9	7	<sup>6</sup> 8	2
G	3	9	1	4	<sup>5</sup> <sub>8</sub>	6	<sup>2</sup> <sub>5</sub> 8	<sup>2</sup> <sub>5</sub>	7
H	4	7	6	2	3	<sup>5</sup> 8	<sup>5</sup> <sub>8</sub>	1	9
J	<sup>5</sup> 8	<sup>2</sup> <sub>5</sub>	<sup>2</sup> <sub>5</sub> 8	9	7	1	6	4	3

In Simple Colouring are looking for pairs of a number in any row, column, box or diagonal. When you spot them you alternate the colors. Thus C1 is red, it can see C3 and J3 which are coloured blue. These cells can see others - note the blue to red link between C3 and G7.

All of red or all of blue must be correct. Since we have two red cells that can see each other (D6 and G6) then all reds must be wrong and we can remove them.

Thanks to Ruud for pointing out the solution to this puzzle.

	1	2	3	4	5	6	7	8	9
A	2 3 8	2 4 8	1 3	5	9	7	1 3 8 6	4 3 8 6	4 3 8
B	3 5 8 9	4 8	1 3 9	6	4 8	2	1 3 8	4 3 8 9	7
C	3 5 8 9	7	6	4 3 8	4 3 8	1	2	4 5 3 8 9	4 3 8 9
D	1	9	7	4 3 8	4 3 8 6	4 6	3 8	2	5
E	6	5	2	3 8	1	9	4	7	3 8
F	4	3	8	7	2	5	9	1	6
G	2 8	1	5	9	4 6 8	4 6	7	3 8	2 3 8
H	7	6 4 8 9	2	5	3	8 6 4 9	4 9	1	
J	3 9	2 6 4	3 9	1	7	8	5 6	4 5 6 4 9	2 9

## The Unique Rectangle - pitfalls

The Unique Rectangle is a popular strategy in sudoku - if you are happy to accept the compiler's word that there is only one solution. The concept is to look for four cells in exactly two rows, two columns and two boxes. There are many variations but the principle is to look for a potential "deadly rectangle" which contains swappable pairs of numbers. If there is only one solution then we cant allow four such cells to contain two numbers in total that can be swapped around.

This example is the classic UR - see the three yellow cells and the one red one. In a normal sudoku we could not allow 4 and 6 to remain in D5 since to do so would mean 4 and 6 were swappable between those cells. However, D6 is on a diagonal which means there is an additional reason why that one will turn out to be either 4 or 6. Because of that our uniqueness logic collapses and we cant with any authority remove the 4/6 from D5.

	1	2	3	4	5	6	7	8	9
A	9	6	2 5	7	3	1 5	1 2	4	8
B	3	4	2 5	9	1 5	8	1 2 6	2 6	7
C	8	7	1	2	4	6	3	5 9	5 9
D	1	8	6	5	7	4	2 9	3	2 9
E	2 5	3	7	8	2 6	9	5 6	1	4
F	2 5	9	4	1	2 6	3	7	8	5 6
G	4	1 2	9	6	1 5	7	8	2 5	3
H	6	5	8	3	9	2	4	7	1
J	7	1 2	3	4	8	1 5	2 5 6 9	2 5 6 9	2 6

## Almost Avoidable Rectangle pitfall

In a normal sudoku we can say something about any four cells that were not clues but have three worked out. If the three known cells look like they form a swappable pair with the fourth unknown cell (that has two or more candidates on it) then none of those know solutions can appear in the unknown cell. We want to avoid a swappable rectangle.

A1, A9, C1 and C9 look like an Avoidable Rectangle since 9 is an option for C9 - but this is not a normal sudoku so 9 cannot be removed from that cell. The reason is that this AAR contains cells on the diagonals. Because the diagonals have their own constrains the 8 and 9 are not infact swappable and there is only one solution. This means it is not valid to use this strategy in this case. It is however useful on cells that are not on the diagonal.

# SOLUTIONS TO EXAMPLES

**1.**

3	1	8	7	2	4	9	6	5
4	2	7	6	9	5	8	1	3
9	6	5	1	8	3	4	7	2
1	8	4	9	3	6	2	5	7
2	9	3	5	7	1	6	8	4
5	7	6	2	4	8	3	9	1
7	5	9	4	6	2	1	3	8
6	3	2	8	1	7	5	4	9
8	4	1	3	5	9	7	2	6

**2.**

2	6	4	1	8	9	7	3	5
5	8	7	4	6	3	9	1	2
3	1	9	7	5	2	8	4	6
7	3	1	5	2	4	6	8	9
4	9	5	6	3	8	1	2	7
8	2	6	9	1	7	3	5	4
1	5	2	3	7	6	4	9	8
9	7	8	2	4	1	5	6	3
6	4	3	8	9	5	2	7	1

**3.**

2	5	4	3	6	1	9	8	7
3	7	6	4	8	9	1	2	5
8	1	9	7	2	5	3	6	4
4	6	2	1	7	8	5	3	9
9	3	7	5	4	2	6	1	8
5	8	1	6	9	3	7	4	2
6	2	5	9	3	4	8	7	1
7	9	8	2	1	6	4	5	3
1	4	3	8	5	7	2	9	6

**4.**

3	7	2	9	1	8	5	6	4
4	9	6	3	2	5	7	8	1
5	8	1	6	4	7	3	2	9
9	4	5	2	3	1	8	7	6
2	6	8	4	7	9	1	3	5
1	3	7	5	8	6	9	4	2
7	5	9	8	6	2	4	1	3
8	2	3	1	9	4	6	5	7
6	1	4	7	5	3	2	9	8

**5.**

1	8	6	4	3	2	9	7	5
3	4	5	1	9	7	6	2	8
9	2	7	5	6	8	3	4	1
4	7	9	2	5	1	8	3	6
5	1	2	6	8	3	4	9	7
8	6	3	7	4	9	1	5	2
7	3	4	8	2	6	5	1	9
2	9	8	3	1	5	7	6	4
6	5	1	9	7	4	2	8	3

**6.**

5	1	8	9	3	2	7	6	4
7	3	2	6	5	4	8	9	1
4	9	6	1	7	8	5	3	2
1	5	7	4	2	6	3	8	9
2	4	9	5	8	3	1	7	6
8	6	3	7	1	9	4	2	5
6	7	1	8	9	5	2	4	3
9	2	5	3	4	7	6	1	8
3	8	4	2	6	1	9	5	7

5	1	4	3	2	7	8	9	6
8	3	2	9	6	5	7	1	4
7	6	9	4	8	1	2	3	5
3	8	5	2	1	9	6	4	7
6	9	1	5	7	4	3	2	8
2	4	7	8	3	6	1	5	9
9	7	3	1	5	8	4	6	2
1	5	6	7	4	2	9	8	3
4	2	8	6	9	3	5	7	1

7.

8.

6	2	7	4	1	3	5	8	9
4	1	8	7	9	5	2	3	6
3	5	9	8	6	2	7	4	1
7	8	2	3	5	1	6	9	4
5	3	6	9	2	4	8	1	7
1	9	4	6	7	8	3	5	2
9	6	5	1	8	7	4	2	3
2	4	1	5	3	6	9	7	8
8	7	3	2	4	9	1	6	5

9	2	1	5	8	6	3	4	7
7	6	3	4	9	2	5	8	1
4	8	5	1	3	7	2	9	6
8	1	2	3	6	4	7	5	9
6	9	4	7	1	5	8	2	3
3	5	7	9	2	8	1	6	4
1	7	6	2	5	9	4	3	8
2	3	9	8	4	1	6	7	5
5	4	8	6	7	3	9	1	2

9.

The so-called 'unsolveables' are left to the reader